Why radians?

Q: Why should we put in all this effort to learn to describe angles in radians, when we've been using degrees just fine our whole lives? What's wrong with degrees?

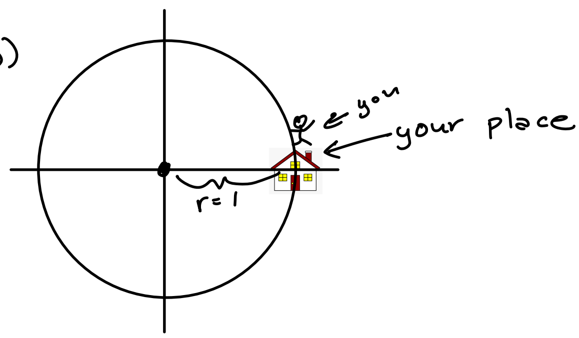
A: Nothing... except that we (humans) made them up. Chop up a circle in 360 pieces (as you would if you were an ancient Babylonian astronomer with a base-60 counting system...) and each one is a degree. (Chop up a circle in 400 pieces, as you would if you were a modern person with a base-10 counting system, and each one is a gradian... now that you know that feel free to forget it.)

But those all depend on choosing some number of pieces to chop up your circle in. Radians don't depend on a choice. They're already there in the circle...

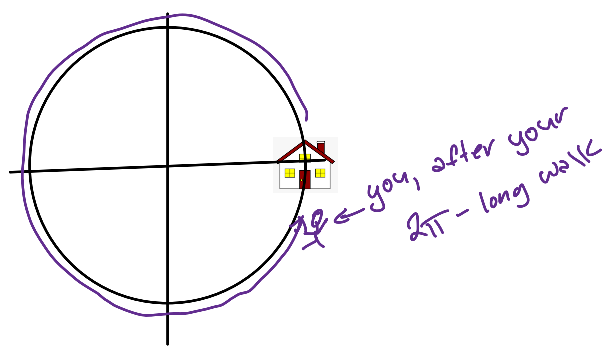
We know the number $\pi = \frac{\text{circumference}}{\text{diameter}}$ of a circle

$\rightarrow \pi = \frac{C}{2r} \rightarrow \boxed{C = 2\pi r}$

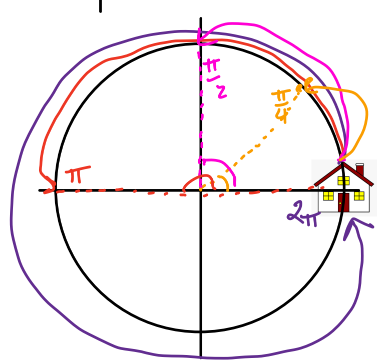
So if you live at position (1,0) on circle of radius = 1 (which, by convention, you do...)



If you go for a walk all the way around the circle, by convention doing this counter-clockwise, then by definition you have walked 2π.



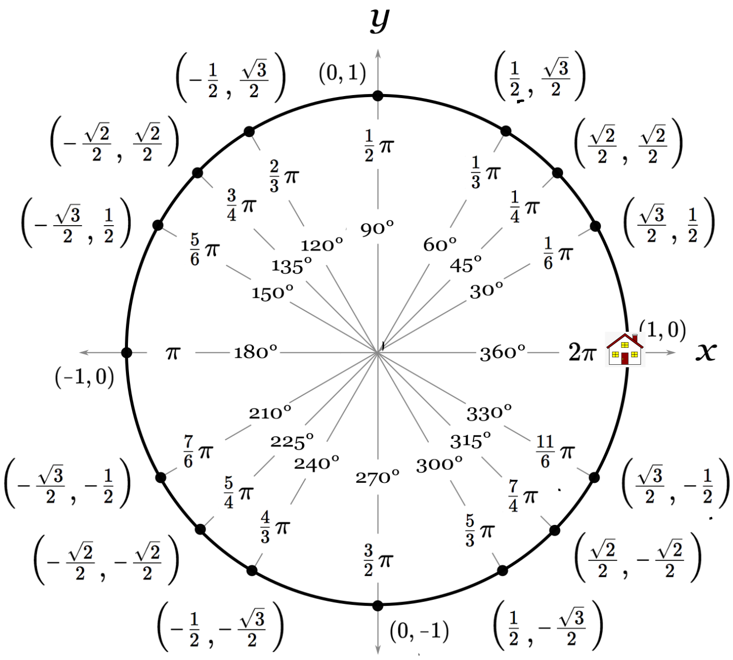
And if it makes sense to go for a 2π - long walk, surely it also makes sense to go for a π walk ... and a $\frac{1}{2}\pi$ walk... etc... so the lengths of walks are not just lengths, they are also locations... and thus angles.



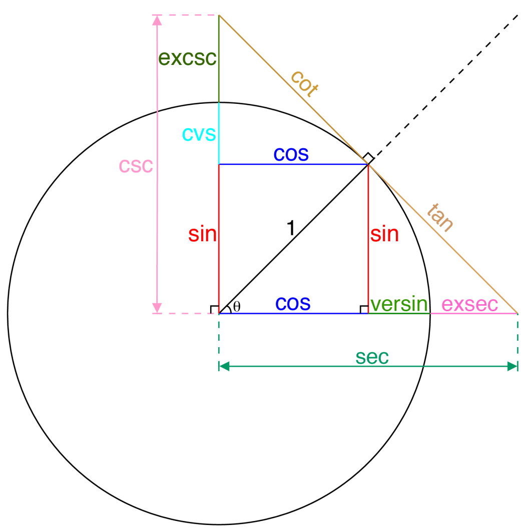
So we now have an entire system of describing angles— *without* having made any arbitrary, human choices about how many pieces to chop the circle in, but only using information provided *by the circle itself.* Which means that if an alien landed in Regina tomorrow, and you needed to figure out how to find some kind of common ground between the system of symbols we use on earth to communicate meaning, and whatever kind of language or communication the aliens use... if you drew them a circle and started labelling it, that alien would immediately know the meaning of the symbols you were showing them, and could show you what symbols they use for the same things... because they know about radians too, just from thinking about circles.

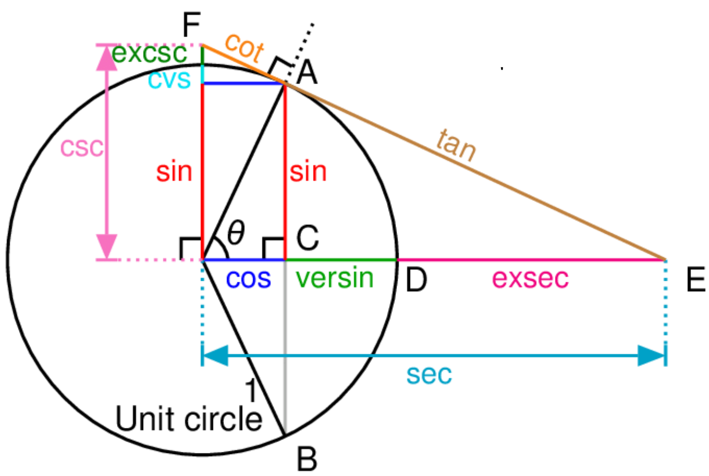
More visuals:

A unit circle with (x,y) = (cosθ, sinθ) coordinates at each important angle:

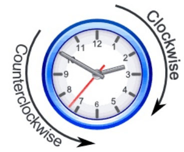


Two different versions of the same image; in the second, the angle has simply moved a bit farther on its "walk around the unit circle". You can forget most of the more exotic functions, but they're here if you want them...





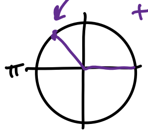
(Psst... been a while since you read one of these?
Counterclockwise = positive
Clockwise = negative)



Convert the given angles from degree to radian or vice versa:

Degree	Radian
135°	$\frac{3}{4}\pi$ rad
-20°	$-\frac{1}{9}\pi$ rad
36°	π/5
210°	7π/6

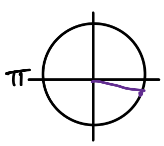
a) 135°



so we should expect the fraction beside π in rad form to be less than 1.

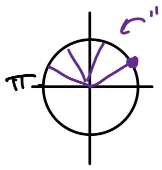
$135 \cdot \frac{\pi}{180} \rightarrow \frac{3}{4}\pi$ rad

b) -20°



$\rightarrow -20 \cdot \frac{\pi}{180} \rightarrow -\frac{1}{9}\pi$ rad

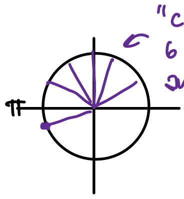
c) $\frac{1}{5}\pi$



"chop π in 5 pieces"

$\rightarrow \frac{1}{5}\pi \cdot \frac{180}{\pi} = 36^\circ$

d) $\frac{7}{6}\pi$

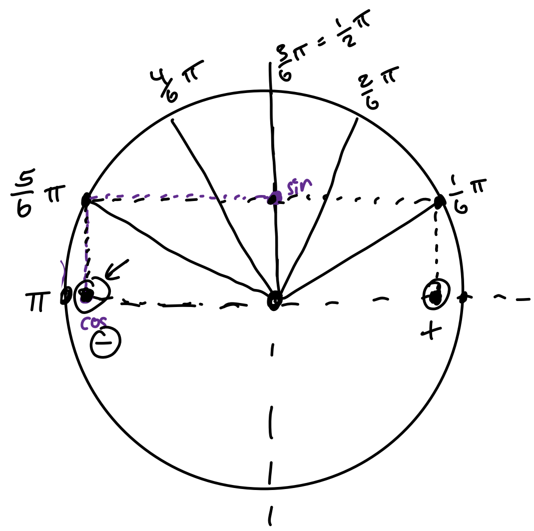


"chop π in 6 pieces, add another of the same size"

$\rightarrow \frac{7}{6}\pi \cdot \frac{180}{\pi} = 210^\circ$

Use the unit circle and the table of special angles to find the values of

$$\sin\left(\frac{5\pi}{6}\right) \quad \text{and} \quad \cos\left(\frac{5\pi}{6}\right)$$



From table:

	$\cos\theta$	$\sin\theta$
$\frac{1}{6}\pi =$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

$$\sin\left(\frac{5}{6}\pi\right) = \sin\left(\frac{1}{6}\pi\right)$$

$$\rightarrow \sin\left(\frac{5}{6}\pi\right) = \frac{1}{2}$$

$$\cos\left(\frac{5}{6}\pi\right) = -\cos\left(\frac{1}{6}\pi\right)$$

$$\rightarrow \cos\left(\frac{5}{6}\pi\right) = -\frac{\sqrt{3}}{2}$$

In the previous question, you found that $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

a) Now find <u>all</u> angles θ such that $\cos(\theta) = -\frac{\sqrt{3}}{2}$

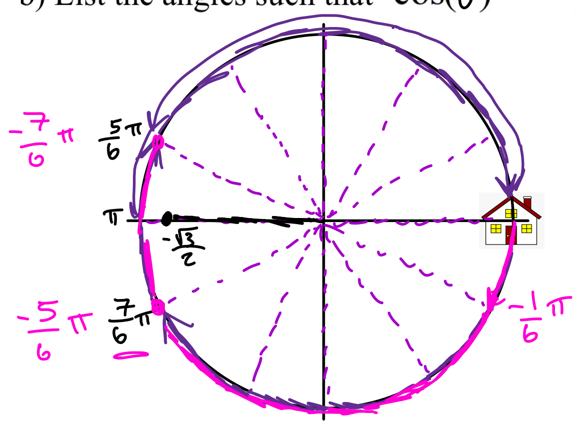
already found one : $\theta = \frac{5\pi}{6}$

Looking for the other location on the circle with an x-coordinate of $-\frac{\sqrt{3}}{2}$. So go down until you see the other: $\frac{7}{6}\pi$

$$\left.\begin{aligned} \theta &= \tfrac{5}{6}\pi + 2k\pi \\ \theta &= \tfrac{7}{6}\pi + 2k\pi \end{aligned}\right\} \text{ where } k \in I$$

← this just means... "you can get back to this location by walking all the way (2π) around the circle, any integer (k) number of times you want."

b) List the angles such that $\cos(\theta) = -\frac{\sqrt{3}}{2}$ that are in the range $\theta \in [-2\pi, \pi]$



You go on a negative, i.e clockwise, walk around the unit circle. The first point you hit is $-\frac{5}{6}\pi$, then $-\frac{7}{6}\pi$. Then, you go on a positive, i.e counterclockwise, walk to π, and hit $\frac{5}{6}\pi$ on the way.

Since we're thinking of the circle as being chopped up into slices of $\frac{1}{6}\pi$ at the moment...

$[-2\pi, +\pi]$ is like saying $[-\frac{12}{6}\pi, +\frac{6}{6}\pi]$ Within this range, the angles that have a cos (x-coor) of $-\frac{\sqrt{3}}{2}$ are indeed $-\frac{7}{6}\pi$, $-\frac{5}{6}\pi$, and $\frac{5}{6}\pi$.

Solve the trigonometric equation $2\sin(x)\cos(x)+\cos(x)=0$ for x.

$$2\sin(x)\cos(x) + \cos(x) = 0 \longrightarrow \cos(x)\left[2\sin x + 1\right] = 0$$

$$\cos(x) = 0 \longrightarrow \frac{1}{2}\pi, \quad \frac{3}{2}\pi$$

$$2\sin(x) + 1 = 0 \longrightarrow \sin(x) = -\frac{1}{2} \longrightarrow \frac{7}{6}\pi, \quad \frac{11}{6}\pi$$

$$\left.\begin{aligned} x &= \frac{1}{2}\pi + 2k\pi \\ x &= \frac{3}{2}\pi + 2k\pi \\ x &= \frac{7}{6}\pi + 2k\pi \\ x &= \frac{11}{6}\pi + 2k\pi \end{aligned}\right\} k \in I$$