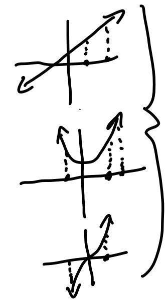
"So this functions business - are we just replacing "y" with "f(x)" in the equations we already know?"

Sometimes!

$y = x+3 \longrightarrow f(x) = x+3$ ✓

$y = x^2+3 \longrightarrow f(x) = x^2+3$ ✓

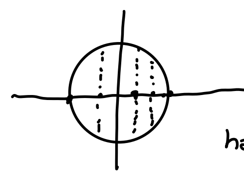
$y = x^3 \longrightarrow f(x) = x^3$ ✓



one input gives one output

etc. But recall that what's implied by the notation of a function is that the equation is a "machine" for transforming input into output. A function can only have one output for each input. So can I replace y with f(x) in a circle?

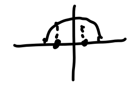
$x^2+y^2=1 \xrightarrow{???} x^2+[f(x)]^2=1$ × no!



"almost all" inputs in the circle's domain have two outputs

"Just watch me... I could make that a function like the others! I'll do this..." $x^2+[f(x)]^2=1 \rightarrow [f(x)]^2=1-x^2 \longrightarrow f(x)=\sqrt{1-x^2}$

But the $\sqrt{\ }$ symbol returns the principal square root... so this is a function, but only because you've chopped off the bottom half of the circle!



Graphs of Functions: **Intercepts and Symmetry** (Lab Manual Unit 11)

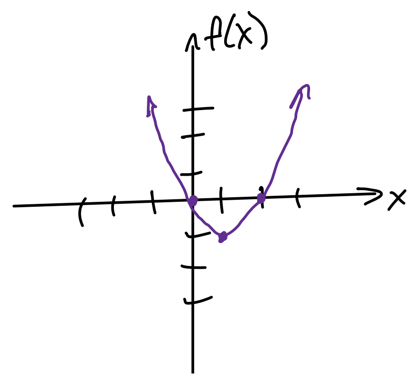
Our concepts of the Graphing Unit carry over to Functions.

The graph of a function is constructed by assigning the function output to the y-axis of the graph, i.e. $y = f(x)$.

eg) Sketch the graph of $f(x) = (x-1)^2 - 1$

We need vertex and intercepts.

$f(x) = (x-1)^2 - 1$ $V(1,-1)$



y-int i.e. where $x=0$

$f(0) = (0-1)^2 - 1 \rightarrow f(0) = (-1)^2 - 1$

$\rightarrow f(0) = 0$

x-int where $f(x)=0 \rightarrow 0 = (x-1)^2 - 1 \rightarrow 0 = (x-1)(x-1) - 1$

$\rightarrow 0 = x^2 - 2x \rightarrow 0 = x(x-2) \rightarrow \boxed{x=0}$ $\boxed{x=2}$

Intercepts:

To find the y-intercept of a function $y = f(x)$, we set x=0.

Note: unlike other equations, functions can have at most one y-intercept. They may also have no y-intercept if the point x=0 is not in the function's domain.

To find x-intercepts of a function $y = f(x)$, we set $f(x) = 0$ and solve for x.

eg) Find the x- and y-intercepts of

a) $f(x) = \dfrac{x^2 - 4}{x^2 + 1}$ b) $f(x) = \dfrac{x^2 + 1}{x}$

a) $f(x) = \dfrac{x^2 - 4}{x^2 + 1}$

x-int: where $f(x) = 0$

$0 = \dfrac{x^2 - 4}{x^2 + 1} \rightarrow 0 = x^2 - 4 \rightarrow 0 = (x+2)(x-2)$

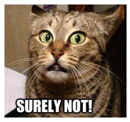
$\rightarrow \boxed{\begin{array}{l} x = 2 \\ x = -2 \end{array}}$

y-int : where $x = 0$

$f(0) = \dfrac{0^2 - 4}{0^2 + 1} \rightarrow f(0) = -\dfrac{4}{1} \rightarrow \boxed{y = -4}$

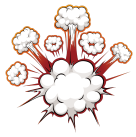
b) $f(x) = \dfrac{x^2 + 1}{x}$ D: $x \neq 0$

x-int: where $f(x) = 0 \rightarrow 0 = \dfrac{x^2+1}{x} \rightarrow 0 = x^2 + 1$

$\rightarrow x^2 = -1$ 

$\therefore$ no real x-intercept

y-int: where $x=0$

$f(0) = \dfrac{0^2+1}{0}$  $\therefore$ no y-intercept

what do we expect this to look like?

- we know it won't touch either of the axes
- We know the upstairs will always be positive, and the downstairs will match the sign of the input x; so, overall the output has the same sign as input, so it uses quadrants 1 and 3
- What will happen around $x=0$? We can't plug in 0, but we can plug in stuff that's pretty darn close...

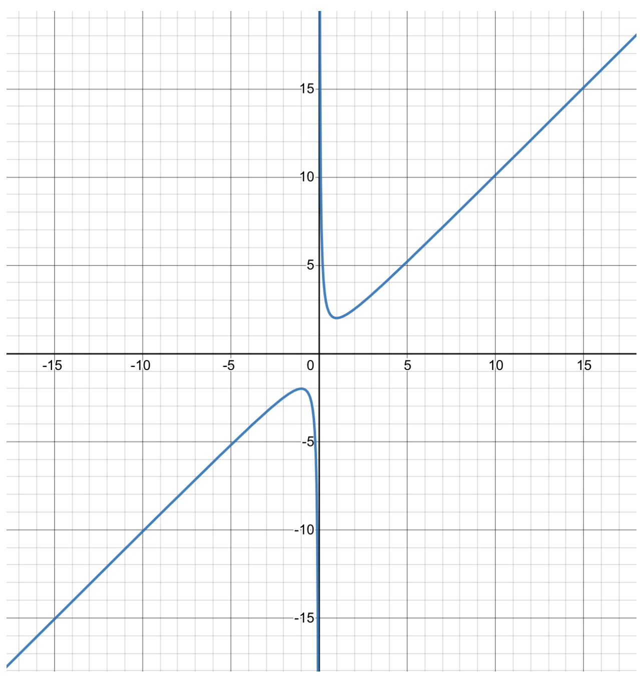
$\dfrac{1}{0.1} = 10$ $\quad \dfrac{1}{0.001} = 1000$ $\quad \dfrac{1}{0.0000...1} = 100000....$ keeps getting bigger!

So we expect that, as we get closer to 0 from the positive (right) side, the output goes towards $+\infty$

Similarly, $\dfrac{1}{-0.1} = 10$ $\quad \dfrac{1}{-0.001} = -1000$ $\quad \dfrac{1}{0.0000...1} = -100000...$

And we expect that, as the input gets closer to 0 on the neg. (left) side, the output goes towards $-\infty$

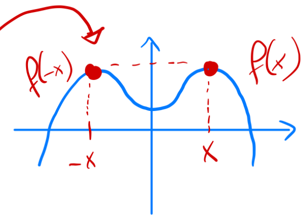
Our expectations are met: those vertical-looking lines will never actually touch the y-axis.



Symmetry:

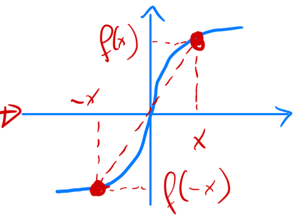
Functions can be symmetric across the y-axis.
We call such functions "even".
We test "even" symmetry by checking if

$$f(-x) = f(x).$$

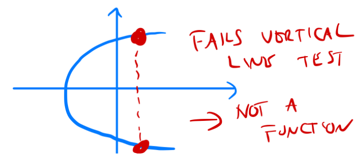


Functions can also be symmetric across the origin.
We call such functions "odd".
We test "odd" symmetry by checking if

$$f(-x) = -f(x).$$



Note that function cannot ever by symmetric across the x-axis. (Why?)



Eg) Test for symmetry:

a) $f(x) = x^4 - 3x^2 + 1$

b) $f(x) = \dfrac{x}{x^2 - 4}$

a) $f(x) = x^4 - 3x^2 + 1$

$f(-x) = (-x)^4 - 3(-x)^2 + 1$

$\rightarrow f(-x) = x^4 - 3x^2 + 1$ $\quad \therefore$ is even

b) $f(x) = \dfrac{x}{x^2 - 4}$

$f(-x) = \dfrac{(-x)}{(-x)^2 - 4} \rightarrow f(-x) = -\left[\dfrac{x}{x^2 - 4}\right]$ $\quad \therefore$ is odd

Evenness/oddness is a special property that "most" functions don't have... if you closed your eyes and draw a function at random, you'd be pretty unlikely to accidentally draw a perfectly symmetric one!