Solve for x:

a) $\log_3(3x+1) + \log_3(3x-1) = 1$ (Similar to #4 on Webwork assignment)

$$\log_3(3x+1) + \log_3(3x-1) = 1$$

$$\rightarrow \log_3\left[(3x+1)(3x-1)\right] = 1$$

$$\rightarrow \log_3(9x^2-1) = 1$$

recall: $\log_a(a^x) = x$ and $a^{\log_a(x)} = x$

$$3^{\log_3(9x^2-1)} = 3^1 \rightarrow 9x^2-1 = 3 \rightarrow 9x^2 = 4 \rightarrow x^2 = \frac{4}{9}$$

$$x = \pm\sqrt{\frac{4}{9}} \rightarrow x = \pm\frac{2}{3}$$

$\rightarrow$ $x = \frac{2}{3}$

$x = -\frac{2}{3}$ ← can't have negative inputs to logs, so this is out!

$$\rightarrow \boxed{x = \frac{2}{3}}$$

b) $e^{2x} + e^{x} = 2$ (Similar to #9 on Webwork)

$$e^{2x} + e^{x} = 2 \longrightarrow (e^{x})^2 + e^{x} = 2$$

This is just a quadratic where you need to solve for e^x, like any other variable. You can either just solve for it directly, or if you find it visually confusing, substitute a different variable for e^x - but <u>don't</u> just replace e^x with x, because $e^x = x$ is untrue and you're going to confuse yourself by changing the definitions of variables. If you want to substitute a new, nicer-looking variable, choose a new letter - and then remember to substitute back at the end and solve for x. I'll use m:

let $e^x = m$

$$\rightarrow m^2 + m = 2 \rightarrow m^2 + m - 2 = 0$$

$\cdot$ to -2 $\quad 2, -1$
$+$ to 1

$$\rightarrow (m+2)(m-1) = 0$$

$m = -2 \rightarrow e^x = -2$ impossible!

$m = 1 \rightarrow e^x = 1$

$$e^x = 1 \rightarrow \ln(e^x) = \ln(1)$$

$$\rightarrow x = \ln(1) \rightarrow \boxed{x = 0}$$

A population is growing exponentially. The population grows by 8.2% each year.

a) Find the continuous growth rate 'r' in the model equation $N(t) = N_0 e^{rt}$.
b) Find the doubling time (i.e. the time in which the population size doubles).
c) The population was 6 million in 2018. What was the population in 2010?

Two important, equivalent ways of writing this:

Yearly model: $N(t) = N_0 (a)^t$
↳ N_0 "naught": amount at $t=0$

Continuous model: $N(t) = N_0 (e^r)^t \rightarrow N(t) = N_0 e^{rt}$

I'll start out by writing the info in the question as $N(t) = N_0 (a)^t$. The base a is the amount the pop. is multiplied by each year.

What does 8.2% growth mean?
- At the beginning of each year, pop. is 100% its starting size for the year.
- At the end of the year, pop. is 108.2% its starting size for the year.

108.2% → Pop. is multiplied by 1.082 each year.

$N(t) = N_0 (1.082)^t$ "yearly model" equation

Convert into "continuous model" i.e base e

$e^r = 1.082 \rightarrow \ln(e^r) = \ln(1.082)$

$\rightarrow r = \ln(1.082)$ or: $r = 0.0788$

$\boxed{N(t) = N_0 e^{\ln(1.082)t}}$ $\boxed{N(t) = N_0 e^{0.0788t}}$

b) when $t=0$ $N(t) = N_0$
what is t when $N(t) = 2N_0$

$\frac{2N_0}{N_0} = \frac{N_0 e^{0.0788t}}{N_0} \rightarrow 2 = e^{0.0788t} \rightarrow \ln(2) = \cancel{\ln}(\cancel{e}^{0.0788t})$

$\rightarrow 0.0788t = \ln(2) \rightarrow t = \frac{\ln(2)}{\ln(1.082)} \rightarrow t \approx 8.8$

c) If 2010 is $t=0$ 2018 is $t=8$, and we're looking for the value of N_0 (aka $N(0)$.)

$N(8)=6$ $\boxed{N(t)=N_0 e^{0.0788t}}$

$$6 = N_0 e^{(0.0788)(8)} \rightarrow N_0 = \frac{6}{e^{(0.0788)(8)}} \rightarrow N_0 = 3.19 \text{ mil.}$$

Or, 2018 can be $t=0$, 2010 is $t=-8$, and we're looking for the value of $N(-8)$.

$$N(0) = N_0 \rightarrow N(0) = 6 \rightarrow N_0 = 6$$

$$N(-8) = 6e^{0.0788(-8)} \rightarrow N(-8) = 3.19 \text{ mil.}$$

d) Instead of using a base-e (continuous) model, use the base-1.082 (yearly) model to show that you get the same doubling time regardless.

$$N(t) = N_0 (1.082)^t$$

Time when $N(t) = 2N_0$

$$\rightarrow 2N_0 = N_0(1.082)^t \rightarrow \boxed{2=(1.082)^t}$$

two ways to solve for t:

I can use a log with the same base as the exponent to eliminate the base of the exponent:

$$\log_{1.082}(2) = \log_{1.082}(1.082)^t \rightarrow t = \log_{1.082}(2) \rightarrow t \approx 8.8$$

Or, I can use a log of some other base, and use the fact that exponents of log inputs can "hop down" to be coefficients:

$$2 = (1.082)^t \rightarrow \ln(2) = \ln(1.082)^t \rightarrow \ln(2) = t \cdot \ln(1.082)$$

$$\rightarrow t = \frac{\ln(2)}{\ln(1.082)} \rightarrow \boxed{t \approx 8.8}$$

e) Starting with 6 million individuals in 2018, multiply by 1.082 each year to estimate the doubling time and confirm the above answer.

2018: 6

2019: 6.492

2020: 7.024

2021: 7.6

2022: 8.22

2023: 8.8979

2024: 9.628

2025: 10.417

2026: 11.27 ← doubled some time in here, i.e

2027: 12.195 at $t \approx 8.8$.