Discriminant reminder:

$\Delta = b^2 - 4ac$

$\Delta > 0$: 2 real roots

$\Delta = 0$: 1 real root

$\Delta < 0$: no real roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Consider the quadratic equation $y = -2x^2 - 8x - 4$

a) Calculate the Discriminant D. What does this tell you?
b) Find all x- and y- intercepts.
c) Write the equation in the form $y = a(x-b)^2 + c$
d) Use this information to sketch a graph of the equation.

$a = -2 \quad b = -8 \quad c = -4$

a) $\Delta = b^2 - 4ac \rightarrow (-8)^2 - (4 \cdot -2 \cdot -4)$

$\rightarrow 64 - (32) \rightarrow \Delta = 32$ $\quad 32 > 0 \quad \therefore$ two real roots

b) x-int: $\frac{-2x^2 - 8x - 4}{-2} = \frac{0}{-2}$

$\rightarrow x^2 + 4x + 2 = 0 \quad a = 1 \quad b = 4 \quad c = 2$

$x = \frac{-4 \pm \sqrt{16 - (4 \cdot 1 \cdot 2)}}{2} \rightarrow x = \frac{-4 \pm \sqrt{4 \cdot 2}}{2}$

$\rightarrow x = \frac{-4 \pm 2\sqrt{2}}{2} \rightarrow$

$x = -2 + \sqrt{2} \rightarrow -0.59$

$x = -2 - \sqrt{2} \rightarrow -3.41$

y-intercepts are where $x=0$

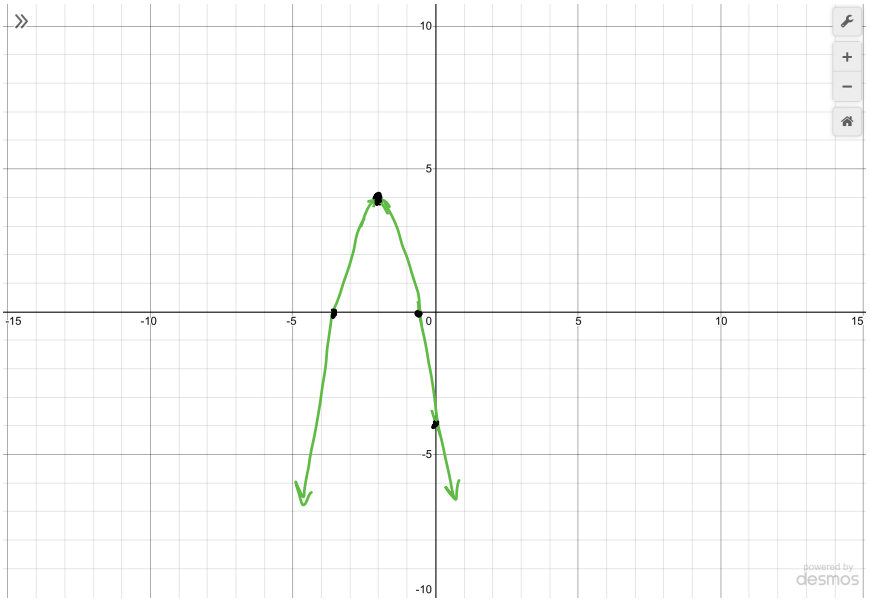
$y = -2x^2 - 8x - 4 \rightarrow \boxed{y = -4}$

c) $y = -2x^2 - 8x - 4 \rightarrow y = -2(x^2+4x) - 4$

↳ $\frac{4}{2} = \boxed{2}$ $2^2 = \boxed{4}$

$\rightarrow y = -2\left[(x+2)^2 - 4\right] - 4 \rightarrow y = -2(x+2)^2 + 4$

$\rightarrow y = -2(x-(-2))^2 + 4$ $\quad V(-2, 4)$



We need to use the vertex, x-intercepts, and y-intercept to plot this.

Describe each of the following graphs in your own words, then sketch the three equations on the same coordinate system:

$y^2 = 4-(x-1)^2, \quad y = 4-(x-1)^2, \quad y = 4-(x-1)$

In general, look at the highest power...

$y^1\ x^1$ → line

$y^1\ x^2$ → parabola
$y^2\ x^2$ → circle
} or x - "stuff" / y - "stuff" squared

$y^2 = 4 - (x-1)^2$ circle

→ $y^2 + (x-1)^2 = 4$ → $(x-1)^2 + (y-0)^2 = 4$

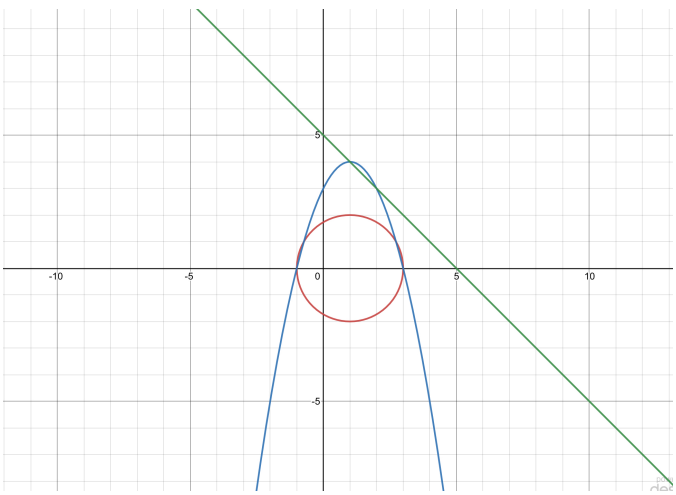
$C = (1, 0)$ $r = \sqrt{4} = 2$ (x-coor of C, y-coor of C, r^2)

$y = 4 - (x-1)^2$ parabola

→ $y = -(x-1)^2 + 4$ opens down $V = (1, 4)$

$y = 4 - (x-1)$ → $y = 4 - x + 1$ → $y = -x + 5$

line $m = -1$ y-int $b = 5$



Some warm-up questions before #3:

Warm-up q. 3.a Find intersection of: $y = x - 5$, $y = 2x + 3$

These are lines, so we expect them to intersect once. The POI is the place where both equations are true at the same time:

$x - 5 = 2x + 3 \rightarrow x - 2x - 5 - 3 = 0 \Rightarrow -x - 8 = 0$

$\rightarrow -x = 8$ $\boxed{x = -8}$ is x-coordinate of point of intersection.

$y = -8 - 5$ $\boxed{y = -13}$ $P(-8, -13)$

Warm-up q. 3b.

Find intersection(s), if any, of $y = x^2 - 5$ and $y = 2x + 3$

A line could intersect a parabola twice, not at all, or - if the line is a "tangent" line - only once. We will set these equal just as before, but we don't know yet whether there are 2, 1 or 0 values of x that can make both true at the same time.

$x^2 - 5 = 2x + 3 \rightarrow x^2 - 2x - 8 = 0$ $a = 1$ $b = -2$ $c = -8$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \dfrac{2 \pm \sqrt{4 - (4 \cdot -8)}}{2}$

positive, so we know we will have two POIs.

$\rightarrow x = \dfrac{2 \pm \sqrt{36}}{2}$ $\rightarrow$ $x = 4 \rightarrow P_1(4, 11)$

$x = -2 \rightarrow P_2(-2, -1)$

3.c Consider the parabola $y = x^2 - 5$ and the line $y = 2x + b$.

Note that the value of the line's intercept, b, is not specified. What value must b take so that the line and the parabola intersect at exactly one point?

$$x^2 - 5 = 2x + b \longrightarrow x^2 - 2x - 5 - b = 0$$

In the previous question, we knew they intersected twice because the discriminant of the quadratic we got by setting the two original equations equal was positive. Here, we know the discriminant has to be 0, because we want to solve for a value that will <u>make</u> it 0.

$$\Delta = b^2 - 4ac = 0$$

$a = 1 \quad b = -2 \quad c = (-5 - b)$

not ideal to have one letter mean two different things... but we'll keep them straight and soldier on.

$$\rightarrow (-2)^2 - [4 \cdot 1 \cdot (-5 - b)] = 0$$

$$\rightarrow 4 - [4(-5 - b)] = 0 \rightarrow 4 - [-20 - 4b] = 0$$

$$\rightarrow 4 + 20 + 4b = 0 \rightarrow 24 + 4b = 0$$

$$\rightarrow \frac{24}{-4} = \frac{-4b}{-4} \rightarrow \boxed{b = -6}$$

$$\boxed{y = 2x - 6}$$