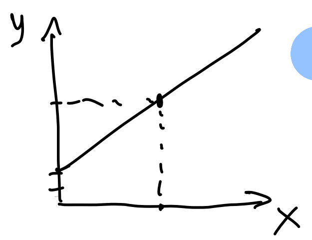
Notation you'll see for lines and linear models...

slope-intercept: $y = mx + b$

point-slope: $y - y_1 = m(x - x_1)$

general/standard: $ax + by + c = 0$

$y = mx + b$ will be most useful for us right now, because it makes clear that the line can act like a little machine: you put in an x-value (independent variable), and get out a y-value (dependent variable), i.e.



$y = x + 2$ → can also be written as $y(x) = x + 2$

Read the LHS as "y, which depends on x, equals..."
or "y, which is a function of x, equals..."
or "y of x equals..."

You can name your independent and dependent variables anything you want... well, almost. Some names are better than others.

If we re-named our dependent variable...

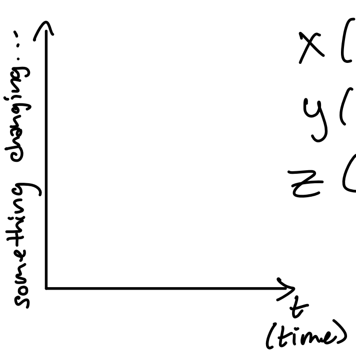
$f(x) = x + 2$, $g(x) = \dots$, $h(x) = \dots$ } good names!

$i(x)$, $j(x)$ } less-good names...

$k(x)$, $e(x)$ } already taken.

$l(x)$, $o(x)$ } Well... these sure are confusingly-shaped letters.

What about if the independent variable was t for time?



$x(t) =$, $y(t) =$, $z(t) =$ } usually means the "output" is the position of a moving object

$T(t) =$ used for Temperature changing with time - yes, two of the same letter?

We are given the line L1 with equation y=3x+2.

a) Find the equation of a second line L2, so that it intersects the first line L1 at the point (x,y)=(1,5) at a 90° angle.

b) A third line L3 has equation 2y-x=4. The three lines form a triangle. Find the coordinates of the vertices of this triangle.

c) Finally, graph the three lines on the same coordinate system.

L1: $y = \frac{3}{1}x + 2$

slope $= \frac{\text{rise}}{\text{run}} \to m = \frac{3}{1}$

y-int $= b = 2$

To graph it, we might also want the x-intercept: $0 = 3x + 2 \to -3x = 2 \to x = -\frac{2}{3}$

a) L2 (1,5) intersects at 90°

Intersect at 90° means slope is negative reciprocal: $m_1 = \frac{3}{1}$ $\boxed{m_2 = -\frac{1}{3}}$

Now, plug in the known (x,y) point and solve for b:

$y = m x + b \to 5 = -\frac{1}{3}(1) + b \to 5 = -\frac{1}{3} + b$

$\to 5 + \frac{1}{3} = b \to \frac{15}{3} + \frac{1}{3} = b \to \boxed{b = \frac{16}{3}}$

L2: $y = -\frac{1}{3}x + \frac{16}{3}$

b) L3 $2y - x = 4 \to \frac{2y}{2} = \frac{x}{2} + \frac{4}{2} \to$ L3: $y = \frac{1}{2}x + 2$

$\boxed{V_1\,(1,5)}$ We already have one vertex, which is the intersection of L1 and L2. Now we need to find two more: the L3/L1 intersection, and the L3/L2 intersection.

L3/L1 Intersection:

$$\underbrace{\text{L3} \quad y = \tfrac{1}{2}x + 2 \qquad \text{L1} \quad y = 3x + 2}_{\text{set equal}}$$

$\frac{1}{2}x+2=3x+2 \rightarrow \frac{1}{2}x-3x=0$ $\boxed{x=0}$

Plug $x=0$ into either equation: $y=3(0)+2 \rightarrow \boxed{y=2}$ $\boxed{V_2\ (0,2)}$

L3/L2 Intersection: $L3 = y=\frac{1}{2}x+2$ $L2\ y=-\frac{1}{3}x+\frac{16}{3}$

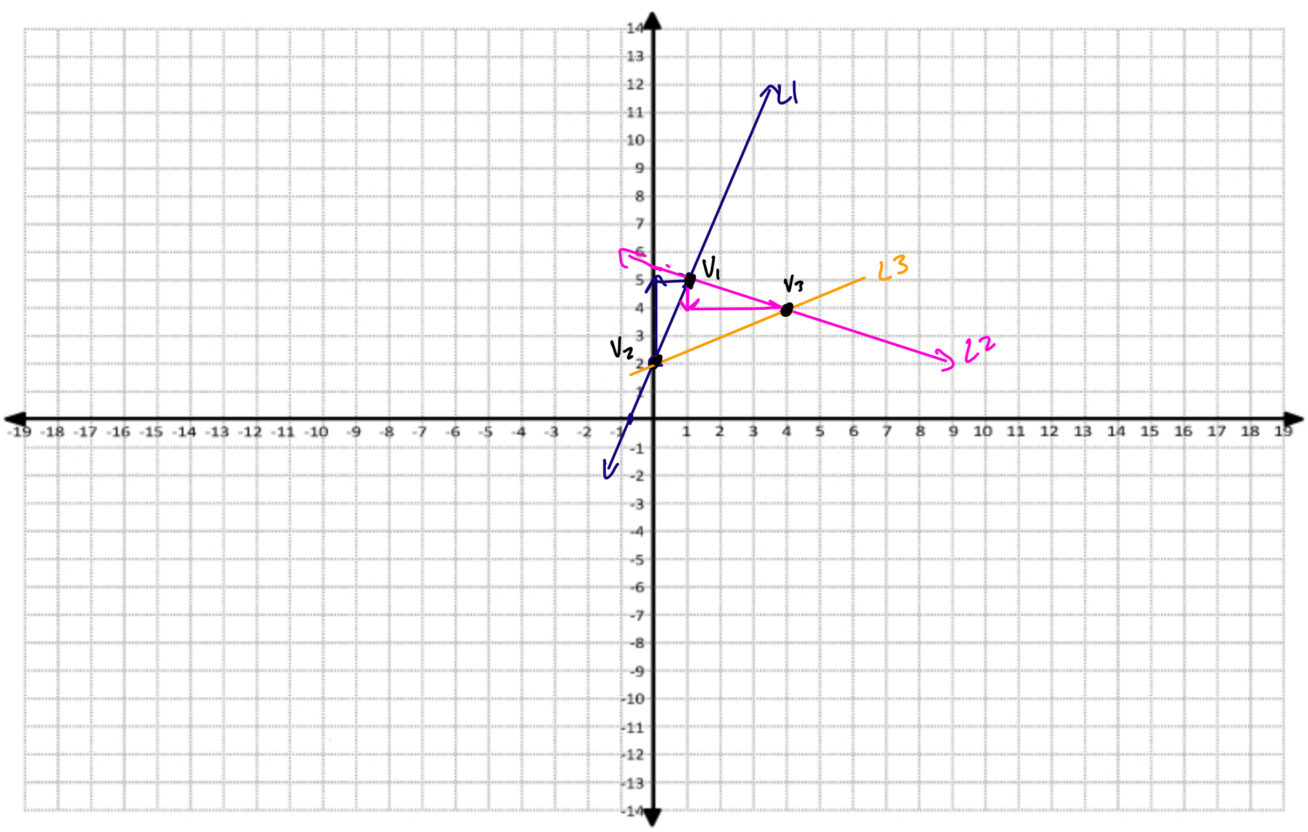
$\frac{1}{2}x+2=-\frac{1}{3}x+\frac{16}{3} \rightarrow \frac{1}{2}x+\frac{1}{3}x=\frac{16}{3}-2$

$\rightarrow \frac{3}{6}x+\frac{2}{6}x=\frac{16}{3}-\frac{6}{3} \rightarrow \frac{5}{6}x=\frac{10}{3}$

$\rightarrow x=\frac{\cancel{10}^{2}}{{}_{1}\cancel{3}}\cdot\frac{\cancel{6}^{2}}{\cancel{5}_{1}} \rightarrow \boxed{x=4}$

$y=\frac{1}{2}(4)+2 \rightarrow y=2+2 \rightarrow \boxed{y=4}$

$\boxed{V_3=(4,4)}$

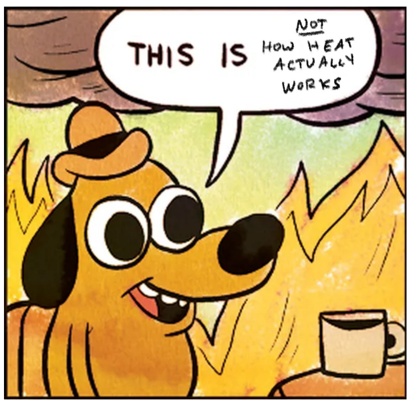


We want to model the temperature of a room "T", exactly "t" minutes after a heater has been turned on.

After 18 minutes, the temperature in the room is 21.2°. Twelve minutes later, the temperature has risen by another 3 degrees. We will assume that the temperature is rising at a constant rate.

a) Find the model equation for T in terms of t.
b) What was the initial temperature in the room?
c) The room is heated to 28°. How long will this take? Give your answer as minutes/seconds.
d) A second room was initially at 30o, and was cooling at a constant rate of one degree per minute. At what time will these two rooms have the exact same temperature?

This question is a linear model and we will solve it as such, but FYI...



$T(t) = mt + b$

$m = ?$
$b = ?$

Slope:

$P_1\ (\underset{x_1}{18}, \underset{y_1}{21.2}) \rightarrow m = \dfrac{y_2 - y_1}{x_2 - x_1} \rightarrow \dfrac{24.2 - 21.2}{30 - 18} \rightarrow \dfrac{3}{12} \rightarrow \boxed{m = \dfrac{1}{4}}$

$P_2\ (\underset{x_2}{30}, \underset{y_2}{24.2})$

But also, we could have read $m = \frac{3}{12}$ right off the question, by noticing that "rise" and "run" are the differences in vertical and horizontal components... which is exactly what "twelve minutes later" and "another three degrees" means.

Now solve for b by plugging in either of your (t, T) points.

$T = \frac{1}{4}t + b \rightarrow 21.2 = \frac{1}{4}(18) + b \rightarrow 21.2 = 4.5 + b$

$\rightarrow \boxed{b = 16.7} \qquad T(t) = \frac{1}{4}t + 16.7 \rightarrow \boxed{T(t) = 0.25t + 16.7}$

b) $T(t) = 0.25t + 16.7$ We want to know the value of T when $t = 0$

$T(0) = 0.25(0) + 16.7 \rightarrow \boxed{T(0) = 16.7}$

c) Now we want to know the value of t when $T = 28$.

$28 = 0.25t + 16.7 \rightarrow 28 - 16.7 = 0.25t$

$\rightarrow \frac{11.3}{0.25} = \frac{0.25t}{0.25} \rightarrow t = 45.2$ minutes

Usually we express time in terms of minutes and seconds, not decimal percentages of minutes. So to know how many seconds 0.2 min is, $60 \cdot 0.2 = 12$

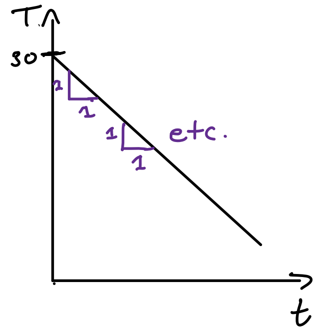
$T = 28^\circ$ at $t = 45$ minutes and 12 seconds.

d) We can read the values of b and m right from the question.

$\boxed{b = 30}$, since that is the value of T at $t = 0$.

The question says that the room cools at a rate of one degree per minute. That means for every one unit of time (run), you go down one unit of temperature (rise).

rise $= -1$, run $= 1$ $\quad m = -\frac{1}{1} \quad \boxed{m = -1}$



Room 2: $T(t) = -t + 30$

To find the intersection with the line modelling Room 1,

$0.25t + 16.7 = -t + 30 \rightarrow 0.25t + t = 30 - 16.7$

$\rightarrow 1.25t = 13.3 \rightarrow t = 10.64 \text{ min} \rightarrow \boxed{10 \text{ min } 38.4 \text{ sec}}$