There are five trigonometric identities that must know. For any angles x and y,

1. $\sin^2(x)+\cos^2(x)=1$
2. $\sin(-x)=-\sin(x)$
3. $\cos(-x)=\cos(x)$
4. $\sin(x+y)=\sin(x)\cos(y)+\sin(y)\cos(x)$
5. $\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$

(2, 3) SYMMETRY → "ODD" / "EVEN"

(4, 5) ADDITION IDENTITIES

Solve the trigonometric equation for all angles x in the interval [-π,2π].

$$2\sin(x)\cos(x)-\sin(x)=0$$

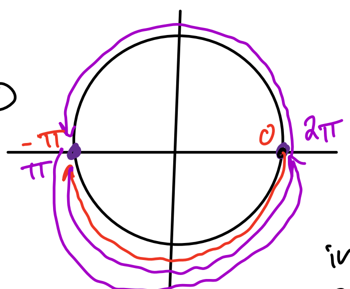
$$2\sin(x)\cos(x)-\sin(x)=0 \rightarrow \sin(x)\left[2\cos(x)-1\right]=0$$

① $\sin(x)=0$

② $2\cos(x)-1=0$

Interval: $[-\pi, 2\pi]$ → "First walk in the negative direction halfway around, then go back and walk all the way around in the positive direction."

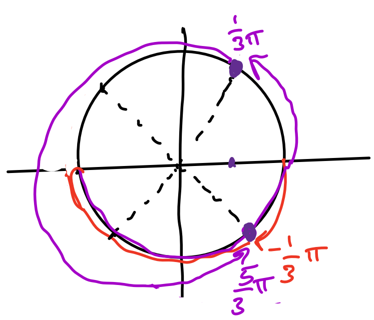
① $\sin(x)=0$



There are two locations on the unit circle with a sine of 0. To find which angles are within our interval, go on a "walk" and note which angles you pass.

$x = 0, -\pi, \pi, 2\pi$

② $2\cos(x)-1=0 \rightarrow \cos(x)=\frac{1}{2}$



$x = -\frac{1}{3}\pi, \frac{1}{3}\pi, \frac{5}{3}\pi$

$\boxed{\text{Values of } x\text{: } 0, -\pi, \pi, 2\pi, -\tfrac{1}{3}\pi, \tfrac{1}{3}\pi, \tfrac{5}{3}\pi}$

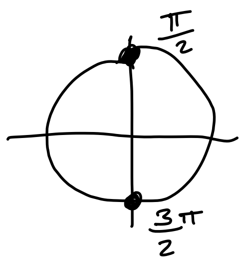
Solve for x: $\underbrace{\cos(2x)} + \sin^2(x) = 0$ $\boxed{\sin^2(x) = [\sin(x)]^2}$

Use the identity: $\cos(x+y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
It just so happens here $y = x$, so $\cos(2x) = \cos^2 x + \sin^2 x$

$$\cos^2(x) - \cancel{\sin^2(x)} + \cancel{\sin^2(x)} = 0$$

$$\rightarrow \cos^2(x) = 0 \rightarrow \cos(x)\cos(x) = 0$$

$$\cos(x) = 0$$



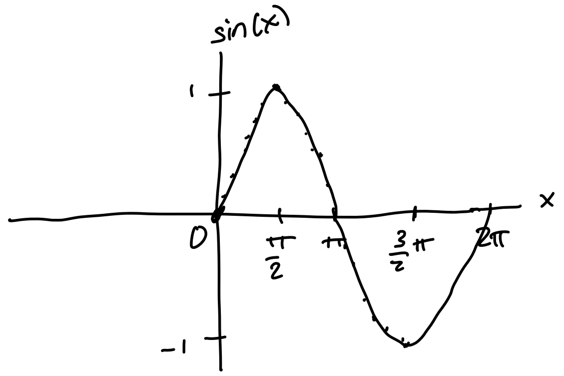
$$\left.\begin{array}{l} x = \frac{\pi}{2} \\ x = \frac{3}{2}\pi \end{array}\right\} + 2k\pi,\ k \in I$$

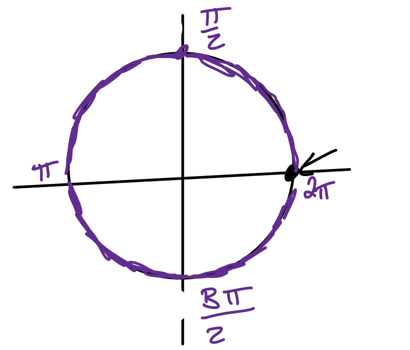
Sketch the graph of $f(x) = 3 - 2\sin(x)$

$f(x) = -2\sin(x) + 3$

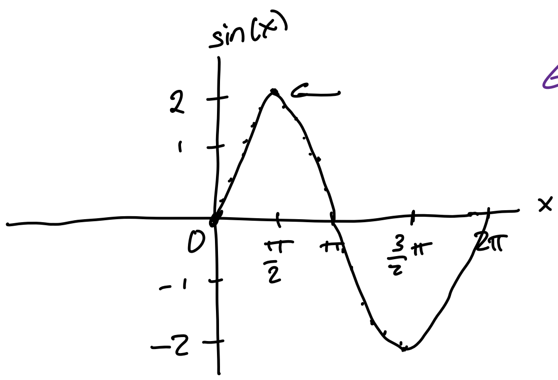
input = location
output = sine of location

$f(x) = \sin(x)$



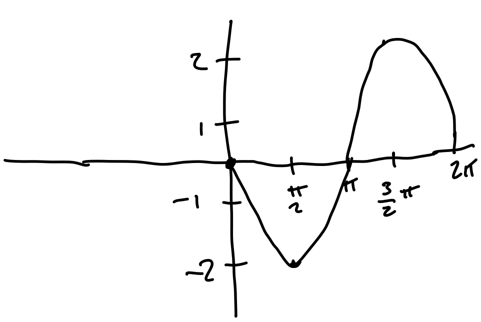


$f(x) = 2\sin(x)$

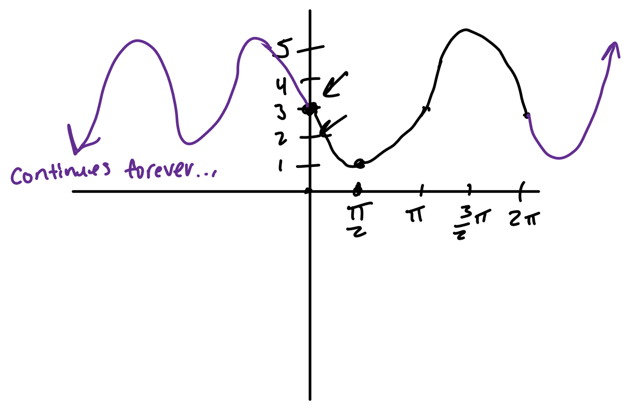


← not to scale compared to the first one...

$f(x) = -2\sin(x)$



$f(x) = -2\sin(x) + 3$



Suppose the temperature (in degrees) as a function of time t (in years) is given by

$$N(t) = 21.4 + 12.7\sin(2\pi(t - 1/3))$$

a) What is the highest and lowest temperature in a year?

b) At what time in the year does temperature reach its maximum?

$$N(t) = 21.4 + 12.7 \sin\left[2\pi\left(t - \tfrac{1}{3}\right)\right]$$

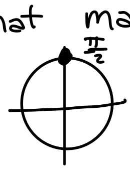
As we saw above... you can add to or multiply the output of a sine function (here, it's being multiplied by 12.7 and having 21.4 added) But the output of sin(anything) will always be between −1 and 1. So whatever complicated stuff you shove in... we always know what the max and min output from sine will be.

a) max temp is where $\sin\left[2\pi\left(t - \tfrac{1}{3}\right)\right] = 1$

$$N(t) = 21.4 + 12.7(1) = 34.1^\circ$$

lowest: $N(t) = 21.4 + 12.7(-1) = 8.7^\circ$

Similarly, we know when that max output of 1 is reached: it's when the input is $\frac{\pi}{2}$.



So to find the input, and then solve for t.

b) $\sin\left[\underbrace{2\pi\left(t - \tfrac{1}{3}\right)}\right] = 1 \rightarrow 2\pi\left(t - \tfrac{1}{3}\right) = \frac{\pi}{2}$

$$2\pi\left(t - \tfrac{1}{3}\right) = \frac{\pi}{2} \rightarrow t - \frac{1}{3} = \frac{\cancel{\pi}}{2} \cdot \frac{1}{2\cancel{\pi}} \rightarrow t - \frac{1}{3} = \frac{1}{4}$$

$\rightarrow t = \frac{1}{4} + \frac{1}{3} \rightarrow t = \frac{7}{12}$ Since t represents years, and there are 12 months, $\frac{7}{12}$ means beginning of July.